# The Semantics of Scientific Theories

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**Abstract**

Marian Przełęcki's semantics for the Received View is a good explication of Carnap's position on the subject, anticipates many discussions and results from both proponents and opponents of the Received View, and can be the basis for a thriving research program.

*Keywords:* semantics of theories; logical empiricism; received view; theoretical terms; vagueness; analyticity; analytic-synthetic distinction

## Contents



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*Dedicated to Marian Przełęcki*
*on occasion of his 90th birthday.*

# 1 Introduction

The Received View in the philosophy of science was the logical empiricists' framework for analyzing theories and related concepts. Developed mainly by Carnap (1939; 1966) and Hempel (1958), and influenced by, for example, Reichenbach (1928), Neurath (1932), Feigl (1956), and Nagel (1961), it arguably formed the core of the logical empiricists' philosophy. In 1969, it stood at a precipice. Carnap had been concentrating on the philosophy of probability for years, without ever having discussed in detail its connection to the Received View.[1] From 1965 through 1969, Hempel had criticized and ultimately abandoned the Received View in a series of talks which were just being published (Hempel 1969; 1970; 1974). Hempel's talk in 1969 played a central role in an influential conference critical of the Received View (Suppe 2000, S102–S103). The proceedings of that conference included what is now widely considered a canonical introduction to the Received View (Suppe 1974a), which recommends its complete rejection. The next years would bring the Received View's spectacular fall into infamy (cf. Lutz 2012b, 79).

In the following, I will argue that this unequivocal dismissal of the Received View after 1969 was far from justified. In fact, while sociologically it stood at a precipice, conceptually it had reached firm ground that could have led—and might still lead—to heights undiscovered by its chronological successors in the philosophy of science. Specifically, I will argue that Marian Przełęcki's outstanding[2] monograph *The Logic of Empirical Theories* and his related articles gave the Received View, for the first time, a natural and precise semantics that can capture major linguistic phenomena encountered in the analysis of scientific theories and provides a unifying framework for discussions in the philosophy of science.[3]

Przełęcki (1969, 1) notes that "a more adequate, though more cumbersome" title of his monograph "would read: the logical syntax and semantics of the language of empirical theories". And Przełęcki (1974b, 402) adds:

> The account of empirical interpretation of scientific theories advanced therein is, as far as I can judge, not a new one; anyway it was not meant to be, as the main purpose of the monograph was to give a brief and elementary account of the current view of the subject. The view presented in the monograph is known under the name of the *Standard* (or *Received*) *View of Scientific Theories*.

[1]Carnap's work on probability may have been directly motivated by the need for probabilistic correspondence rules in the Received View (Lutz 2012b, 109–110).

[2]Two other current philosophers of science have described the book as 'marvelous' and 'wonderful', respectively, so I am in good company.

[3]I have argued elsewhere that the major criticisms of the Received View are spurious (Lutz 2012b; Lutz 2012a, ch. 3, 4).

The logical syntax that Przełęcki lays out is a restriction of Carnap's higher order syntax (cf. Carnap 1939, §§13–19) to first order logic, and thus is indeed not new. The semantics, however, is Przełęcki's own, though informed by the works of Kemeny (1956) and Carnap (1961), which Przełęcki (1969, 107) cites as "classics". I will argue in the following that Przełęcki's semantics completes the Received View in a natural way that fits with Carnap's informal descriptions and arguments (§2), while going beyond Carnap's account both in its precision and in its content. Przełęcki's semantics relies on classical model theoretic notions developed by Tarski and others, but expands them to deal with empirically interpreted theories. It further permits straightforward generalizations without a loss of its basic insights (§3). That Przełęcki's semantics is very natural is indicated by the sheer number of later results he anticipated, coming from both the Received View's proponents *and* its critics (§4). And this treasure trove is far from exhausted. As a simple example of a new result, I will point out an interesting relation between vague languages and analyticity (§5).

The overall picture that will emerge is this: Przełęcki's semantics is a natural formalism for the Received View, and provides a natural framework for understanding the semantics of scientific theories in general. Furthermore, it points the way to significant new research questions and results in the philosophy of science.

## 2 The Received View in the philosophy of science

Over the course of its development, the Received View has seen a variety of formulations and modifications by different authors. In the following, I will focus on Carnap's contributions, and specifically on those that play a major role in Przełęcki's semantics.

### 2.1 The observational-theoretical distinction

The central component of the Received View on scientific theories is its distinction between observational and non-observational (theoretical) sentences. This distinction can be found in Carnap's work from his earliest contribution to the Received View (Carnap 1923, 99–100) to his last (Carnap 1966, ch. 23). In some of his works, the observational and theoretical languages are taken to be completely distinct, where the theoretical language is used as a metalanguage of the observational language, with a translational scheme from observational (object-) sentences to theoretical (meta-) sentences (Carnap 1932, 216–217). However, Neurath (1932, 207) suggested distinguishing between observational and non-observational sentences in *the same* language (cf. Carnap 1932, 215–216);[4] the translational scheme can then be realized by sentences of that same language,

[4]Carnap's conjecture that Neurath was the first to suggest this treatment of the sentences' relation is somewhat puzzling, since Carnap (1928) himself had already used this method.

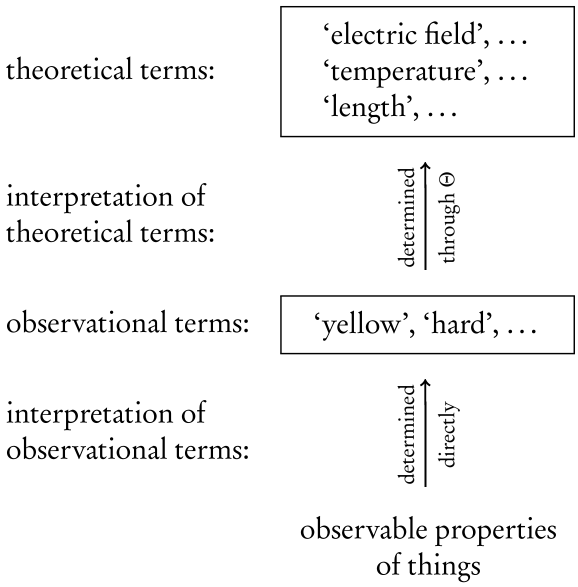


Figure 1: Giving an empirical interpretation to theoretical terms (loosely based on a diagram by Carnap 1939, 205).

known by Carnap (1966, ch. 24) as 'correspondence rules'. One typically makes a distinction between the set $C$ of correspondence rules and the set $T$ of sentences of the theory proper, although for the logical analysis within a single language, it is often convenient to combine them into a set $\Theta \mathrel{\lvert\!=\!\rvert} T \cup C$ of a single new theory.

Given the initial distinction between observational and non-observational sentences, it is typical to distinguish between the set $\mathscr{O}$ of observational terms and the set $\mathscr{T}$ of non-observational (theoretical) terms.[5] Together, they make up the whole of theory Θ's vocabulary $\mathscr{V} = \mathscr{O} \cup \mathscr{T}$.[6] The observational sentences contain only observational vocabulary and may be further restricted in their logical strength, for example to first order logic, finitely quantified first order logic, or molecular sentences (Carnap 1956, 41). Carnap (1963, 959) calls the language whose sentences contain only $\mathscr{O}$-terms but that is otherwise unrestricted the *logically extended observation language*.

The $\mathscr{O}$-terms are directly interpreted, either by observation or by simple, uncontroversial measurements (Carnap 1966, 226–227).[7] The $\mathscr{T}$-terms are not directly interpreted (cf. Carnap 1956, 47) but rather interpreted *only* through the direct interpretation of the $\mathscr{O}$-terms and the relations of the $\mathscr{T}$-terms with the $\mathscr{O}$-terms given by Θ (see figure 1). When interpreting the $\mathscr{T}$-terms one might need to

[5]As is traditional in the philosophy of science, I will use 'term' to refer to any non-logical constant. This fits well with general usage and related terms like 'terminology', but unfortunately not with the usage in symbolic logic.

[6]Throughout, I refer only to non-logical constants as 'the vocabulary of a theory'. Theories and sentences can, of course, always contain logical constants and variable names.

[7]Chang (2005) gives a contemporary defense of such a direct interpretation of $\mathscr{O}$-terms.

introduce new objects that are not observable. Carnap (1958, 237–238, 242–243) assumes that such newly introduced objects can be taken as mathematical objects: Assuming that there are at most countably many observable objects, he suggests mapping them injectively to the natural numbers, and treating theoretical terms as applying only to the natural numbers and objects that can be constructed from the natural numbers with the help of Cartesian products or powersets. Carnap never spelled out this schema in much detail, although it seems that the formalization of a statement like 'Some red object has a temperature of 0°C' would be as follows:

$$\exists x \,.\, Rx \wedge tmx = 0\,,^{8} \tag{1}$$

where $m$ is the mapping from observational objects to natural numbers, $t \in \mathscr{T}$ is the function assigning the temperature in degrees Celsius, and $R \in \mathscr{O}$ refers to red objects. The analogue of the temperature concept of natural language (which assigns a value in degrees Celsius directly to an observable object) is thus not $t$ (which assigns such a value to a natural number), but rather $t \circ m$. Since $m$ is a mapping from observable objects to unobservable ones and its extension can thus not be determined by observation or simple measurement, it is a theoretical term as well. New, unobservable objects are introduced simply as other numbers or more complicated mathematical constructs. Theoretical terms hence apply to them in the same way that they apply to the mathematical representations of observable objects under $m$.[9]

The bipartition between $\mathscr{O}$-terms and $\mathscr{T}$-terms is not fixed. In fact, according to Carnap (1932, 224) one can *choose* the observation terms depending on the context:

> Let $G$ be a law [...]. To check $G$, derive concrete sentences that relate to specific space-time points [...]. From these concrete sentences, derive further concrete sentences using other laws and logico-mathematical rules of derivation, until one reaches sentences that one wishes to accept in the specific case. And it is a matter of choice which sentences one intends to use as these endpoints of the reduction [...]. Whenever one wants to—for instance, if there are doubts or one wants to consolidate the scientific hypotheses more securely—one can reduce those sentences previously accepted as endpoints again to other ones and choose those to be endpoints. *[T]here are no absolute primary sentences* for the construction of science.[10]

[8]Throughout, I assume that quantifiers have minimal scope unless followed by a dot, in which case they have maximal scope.

[9]Carnap introduces this rather circuitous way of interpreting theoretical terms to address the worry that it may be impossible to interpret or discuss unobservable objects or non-observational terms. However, it is at least not obvious why the mapping function $m$ is needed, as Carnap's formalism also works for a mapping $t^*$ that assigns temperatures directly to observational objects (so that $t^* = t \circ m$ for observable objects and $t^* = t$ for unobservable objects).

[10]"Es sei $G$ ein Gesetz [...]. Zum Zweck der Nachprüfung sind aus $G$ zunächst konkrete, auf

This notion of a hierarchy of the scientific language is one of the central assumption of Carnap's account of scientific theories (Lutz 2012a, 124). In figure 1, this assumption could be drawn by partitioning the theoretical terms $\mathscr{T}$ into a series $\mathscr{T}_1, \mathscr{T}_2, \ldots, \mathscr{T}_n$ from lower (more observational) to higher (more theoretical) terms, where the interpretation of the higher terms $\mathscr{T}_k$ is determined solely by the interpretation of the lower terms $\mathscr{T}_{k-1}$ and some theory $\Theta_k$.[11]

## 2.2 The analytic–synthetic distinction

On the basis of a bipartitioned vocabulary, Carnap (1958, 245–246; cf. 1963, 24.D) suggests a distinction between the synthetic (empirical) and analytic component of a theory as follows: Assume that $\Theta$ is a finite conjunction of sentences that describe the theory, and assume that $\Theta = \Theta(O_1, \ldots, O_m, T_1, \ldots, T_n)$, that is, $\Theta$ contains only the $\mathscr{O}$-terms $O_1, \ldots, O_m$ and the $\mathscr{T}$-terms $T_1, \ldots, T_n$.[12] Then

$$\mathsf{R}_{\mathscr{O}}(\Theta) := \exists X_1 \ldots X_n \Theta(O_1, \ldots, O_m, X_1, \ldots, X_n) \tag{2}$$

is $\Theta$'s *Ramsey sentence*, which entails the same $\mathscr{O}$-sentences as $\Theta$ itself. $\Theta$'s *Carnap sentence* is given by

$$\mathsf{C}_{\mathscr{O}}(\Theta) := \mathsf{R}_{\mathscr{O}}(\Theta) \rightarrow \Theta\ . \tag{3}$$

According to Carnap (1963), the Ramsey sentence and the Carnap sentence fulfill the conditions of adequacy for any distinction between the analytic and synthetic component of a theory. To spell out these conditions, Carnap (1963, 963) defines the observational content of any sentence $S$ as follows:

**Definition 1.** The *observational content* or O-content of $S =_{\text{Df}}$ the class of all non-L-true [not logically true] sentences in $L'_{\text{O}}$ which are implied by $S$.

$L'_{\text{O}}$ refers to the logically extended observation language. On this basis, Carnap (1963, 963) suggests

**Definition 2.** $S'$ is *O-equivalent* (observationally equivalent) to $S =_{\text{Df}} S'$ is a sentence in $L'_{\text{O}}$ and $S'$ has the same O-content as $S$.[13]

bestimmte Raum-Zeit-Stellen bezogene Sätze abzuleiten [...]. Aus diesen konkreten Sätzen sind mit Hilfe anderer Gesetze und logisch-mathematischer Schlußregeln weitere konkrete Sätze abzuleiten, bis man zu Sätzen kommt, die man im gerade vorliegenden Fall anerkennen will. Dabei ist es Sache des Entschlusses, welche Sätze man jeweils als derartige Endpunkte der Zurückführung [...] verwenden will. Sobald man will, – etwa wenn Zweifel auftreten oder wenn man die wissenschaftlichen Thesen sicherer zu fundieren wünscht, – kann man die zunächst als Endpunkte genommenen Sätze ihrerseits wieder auf andere zurückführen und jetzt diese durch Beschluß zu Endpunkten erklären. *[E]s gibt keine absoluten Anfangssätze* für den Aufbau der Wissenschaft."

[11]In this particular exposition, Carnap (1939) does not even distinguish between observational and theoretical terms, but rather between elementary and abstract ones. And he notes, for instance, that "if 'iron' is not accepted as sufficiently elementary, the rules can be stated for more elementary terms" (Carnap 1939, 207).

[12]Note that an unadorned '$T$' is a set of sentences (a subset of $\Theta$), while a subscripted '$T_i$' is a theoretical term. This notation is traditional, albeit confusing.

[13]Note that Carnap's definition is asymmetric.

Taking $\mathsf{R}_{\mathcal{O}}(\Theta)$ and $\mathsf{C}_{\mathcal{O}}(\Theta)$ as the first and second components of $\Theta$, respectively, Carnap (1963, 965) states:[14]

> The two components satisfy the following conditions:
>
> (a) The two components together are L-equivalent to $TC\,[:= T \wedge C \mathrel{\models\!\!\!\dashv} \Theta]$.
>
> (b) The first component is O-equivalent to $TC$.
>
> (c) The second component contains theoretical terms; but its O-content is null, since its Ramsey-sentence is L-true in $L'_{\mathrm{O}}$.
>
> These results show, in my opinion, that this method supplies an adequate explication for the distinction between those postulates which represent factual relations between completely given meanings, and those which merely represent meaning relations.

By definition 2, Carnap's condition (b) entails that the first component does not contain $\mathscr{T}$-terms, so that the conditions on a theory's *analytic component* $\mathrm{An}(\Theta)$ and its *synthetic component* $\mathrm{Syn}(\Theta)$ can be formulated as follows:

**Definition 3.** $\mathrm{An}(\Theta)$ is an *adequate analytic component* of $\Theta$ and $\mathrm{Syn}(\Theta)$ is an *adequate synthetic component* of $\Theta$ if and only if

1. $\mathrm{An}(\Theta) \wedge \mathrm{Syn}(\Theta)$ is L-equivalent to $\Theta$,
2. $\mathrm{Syn}(\Theta)$ has the same O-content as $\Theta$,
3. $\mathrm{Syn}(\Theta)$ contains no theoretical terms, and
4. the O-content of $\mathrm{An}(\Theta)$ is the empty set.

As Carnap points out, a sentence's O-content is the empty set if and only if its Ramsey sentence is logically true. This result follows from

**Lemma 1.** *Let $\Phi$ and $\Psi$ be sentences. Then $\Phi$'s O-content is the empty set if and only if $\models \mathsf{R}_{\mathcal{O}}(\Phi)$, and $\Phi$ and $\Psi$ have the same O-content if and only if $\mathsf{R}_{\mathcal{O}}(\Phi) \mathrel{\models\!\!\!\dashv} \mathsf{R}_{\mathcal{O}}(\Psi)$.*

*Proof.* $\Phi$'s O-content is the empty set if and only if all the $\mathcal{O}$-sentences *of any order* that it entails are tautologies (since $L'_{\mathrm{O}}$ contains sentences of any order). Since $\mathsf{R}_{\mathcal{O}}(\Phi)$ is an $\mathcal{O}$-sentence, $\Phi \models \mathsf{R}_{\mathcal{O}}(\Phi)$, and $\mathsf{R}_{\mathcal{O}}(\Phi)$ entails all $\mathcal{O}$-sentences that are entailed by $\Phi$ (Rozeboom 1962, 291–293), this is the case if and only if $\models \mathsf{R}_{\mathcal{O}}(\Phi)$.

[14] These conditions are equivalent to earlier ones that Carnap (1958, 245–246) uses to argue for the adequacy of the Ramsey sentence and the Carnap sentence:

> (2) Jeder Satz ohne $T$-Terme, der aus $TC$ folgt, folgt auch aus $R$. [...]
>
> (3) *a)* Die Konjunktion $R \cdot A_T$ ist $L$-äquivalent mit $TC$.
>
> *b)* Jeder Satz ohne $T$-Terme, der aus $A_T$ folgt, ist $L$-wahr.

In these conditions, Carnap speaks of "sentences without theoretical terms", which, though containing only observational terms, are unrestricted in their logical apparatus.

Two sentences have the same O-content if and only if they entail the same $\mathscr{O}$-sentences of any order. A sentence has the same O-content as its Ramsey sentence, so two sentences have the same O-content if and only if they have equivalent Ramsey sentences. □

Carnap's conditions of adequacy can thus be rephrased:

**Corollary 2.** An(Θ) *is an adequate analytic component of* Θ *and* Syn(Θ) *is an adequate synthetic component of* Θ *if and only if*

1. $\mathrm{An}(\Theta) \wedge \mathrm{Syn}(\Theta) \models\!\!\!\dashv \Theta$,
2. $\mathsf{R}_{\mathscr{O}}(\mathrm{Syn}(\Theta)) \models\!\!\!\dashv \mathsf{R}_{\mathscr{O}}(\Theta)$,
3. Syn(Θ) *contains no theoretical terms, and*
4. $\models \mathsf{R}_{\mathscr{O}}(\mathrm{An}(\Theta))$.

The Ramsey sentence thus allows for a compact description of central empiricist concepts. More importantly, together with the Carnap sentence it fulfills the central need of the logical empiricists' philosophy for an analytic-synthetic distinction.

While the syntactic analysis of scientific theories had at this point in the historical development of the Received View reached a very high level, the Received View's semantics was a mere impressionistic story, not yet described with any formal rigor. It was Marian Przełęcki who first developed and analyzed the semantics of the Received View as rigorously as Carnap did with its syntactic aspect.

## 3 Przełęcki's semantics for the Received View

In his monograph, Przełęcki develops a semantics based on two central assumptions. For one, Przełęcki (1969, 105–106) explicitly sets aside the matter of the *development* of theories. Instead he focuses his analysis on a theory at a specific point in time—the theory's "cross-section"—and considers the development of a theory a series of such cross-sections.[15] This perspective fits nicely with Carnap's and Hempel's analyses, which, although typically referring to a fixed set of sentences of a theory, often assumed that those sentences might change in the future. The best example of this is probably their reliance on conditional rather than explicit definitions in order to allow for the definitions to be strengthened later (Carnap 1936, 449–450; Hempel 1952, 680–681).

More substantially, Przełęcki (1969, 29–30) also assumes that the domain of a theory is fixed in advance in the theory's metalanguage. Przełęcki (1974a, 405)

[15] Przełęcki (1969, 106) provides a nice analogy: "The logical technique resembles here a biological one. Logical reconstruction of a scientific theory is like making 'slices' of a living organism. This certainly distorts our original object of inquiry. But only then can it be put under a logical microscope".

later calls this assumption "unfounded [and] involv[ing] certain undue restrictions", but he is too harsh in judging his assumption unfounded, since in his monograph he refers to Kemeny (1956, 17), who argues that this

> restriction is motivated by the idea that a formal system is the formalization of the abstract structure of a given set of individuals. We can see this in examples from [...] Science. [...] Sociology deals with human beings [and] a sociologist would allow only the set of all human beings (past, present, and future)[.]

Przełęcki (1974b, 405) notes another defense in connection with an argument by Winnie (1967), which I will discuss in the following section. In spite of these two possible defenses, Przełęcki (1973) provides two modifications of his semantics that lift the restriction, which I will discuss in §3.3.

## 3.1 Ostension, vagueness, and approximation

In a central discussion of his monograph, Przełęcki (1969, 24–30) argues that purely verbal means (i. e., the exclusive use of sentences) are insufficient for giving a theory an empirical interpretation. This is justified by by

**Claim 3.** *A set $\Theta$ of sentences cannot determine the domains of its models.*

*Proof (cf. Przełęcki 1969, 30–31).* Let $\mathfrak{A} = \langle|\mathfrak{A}|, R_1, \dots, R_r, f_1, \dots f_s, c_1, \dots, c_t\rangle$ be a model of $\Theta$ and let $B$ be any set with the same cardinality as $|\mathfrak{A}|$. Now define, for any $k$-tuple $\langle x_1, \dots x_k\rangle$ of objects and any function $g$ with domain $|\mathfrak{A}|$, $gR\big(g(x_1), \dots, g(x_k)\big) \Leftrightarrow R_i(x_1, \dots, x_k)$. Then any bijection $g : |\mathfrak{A}| \longrightarrow B$ is an isomorphism from $\mathfrak{A}$ to $\mathfrak{B} = \langle B, gR_1, \dots, gR_r, g \circ f_1, \dots, g \circ f_s, g(c_1), \dots, g(c_t)\rangle$, and thus $\mathfrak{B} \models \Theta$. □

Hence, if all intended interpretations of $\Theta$ are over some domain $A$, $\Theta$ will always also be true of interpretations over some completely unrelated domain $B$. Specifically, $\Theta$ cannot identify any objects in its domain.[16] Even under Przełęcki's restriction of all models of $\Theta$ to the same domain, $g$ may still be any permutation on the domain; hence any object of the domain can be exchanged for any other object of the domain, always leading to another model of $\Theta$ (Przełęcki 1969, 30–31). Relying on the same formal result, Putnam (1989, 353) puts this point in a discussion of his famous model theoretic argument against realism (Putnam 1977) as follows:

> [I]f there is such a thing as 'an ideal theory' [$I$], then that theory can never implicitly define its own intended reference relation. In fact, there are always many different reference relations that make $I$ true, if $I$ is a consistent theory which postulates the existence of more than one object.

[16]This point is closely related to Newman's objection to Russell's theory of perception (Newman 1928, §2).

For this reason, at least *some* terms of $\Theta$ have to be interpreted directly, and alongside Carnap, Przełęcki assumes that these are the $\mathscr{O}$-terms.

As to the means of direct interpretation, Przełęcki (1969, 36–37) suggests *ostension*, the presentation of paradigmatic cases (the *positive standards*) and paradigmatic non-cases (the *negative standards*) of a predicate. By a *psychological* process, a student who is presented with the positive and negative standards can then learn to identify reliably other objects that the teacher considers to fall under the predicate. Importantly, Przełęcki (1969, 37) points out:

> If one chooses the 'right' class [i. e. the one intended by the teacher] as the denotation of the predicate, one is not compelled to this choice by purely logical reasons. This conclusion is arrived at in some process of abstraction whose analysis presents a problem for a psychologist rather than for a logician.

Hence the efficacy of ostension for observational terms is an empirical problem, which lies outside the philosophy of science.[17] Because $\mathscr{O}$-terms are *only* ostensively interpreted there are no analytic $\mathscr{O}$-sentences; otherwise, according to Przełęcki (1969, 37), their interpretation would be partly determined verbally, and thus not by ostension.

A direct interpretation of the $\mathscr{O}$-terms and a fixed domain determine an $\mathscr{O}$-structure $\mathfrak{N}_{\mathscr{O}} := \langle |\mathfrak{N}_{\mathscr{O}}|, O_1^{\mathfrak{N}_{\mathscr{O}}}, \dots, O_m^{\mathfrak{N}_{\mathscr{O}}} \rangle$. The interpretation of the $\mathscr{T}$-terms within $\Theta$ is determined *only* by $\mathfrak{N}_{\mathscr{O}}$ and $\Theta$, just as assumed by Carnap. This is the *Thesis of Semantic Empiricism* (Przełęcki 1974b, 402, 405; cf. Rozeboom 1962). But only a portion of $\Theta$ contributes to the interpretation of the $\mathscr{T}$-terms, namely $\Theta$'s analytic component $\mathrm{An}(\Theta)$. $\Theta$'s synthetic component $\mathrm{Syn}(\Theta)$ rather contains $\Theta$'s empirical claims. Hence any expansion of $\mathfrak{N}_{\mathscr{O}}$ to a model $\mathfrak{N}$ of $\mathrm{An}(\Theta)$ ($\mathfrak{N}|_{\mathscr{O}} = \mathfrak{N}_{\mathscr{O}} \wedge \mathfrak{N} \vDash \mathrm{An}(\Theta)$) provides a possible interpretation of the $\mathscr{T}$-terms (Przełęcki 1969, 48–50). Such an expansion is unique for any $\mathfrak{N}_{\mathscr{O}}$ if and only if $\mathrm{An}(\Theta)$ entails a definition for every $\mathscr{T}$-term (Beth 1953), and so $\mathfrak{N}_{\mathscr{O}}$ and $\mathrm{An}(\Theta)$ typically determine a set of structures for $\mathscr{V}$ rather than a single structure. In an earlier paper, Przełęcki (1964a) considers the resulting sets of structures for a special kind of analytic sentence: conditional definitions of $\mathscr{T}$-terms. As he points out, a conditional definition of $T_i$ leads to a *vague denotation*, for some objects of the domain $|\mathfrak{N}_{\mathscr{O}}|$ will be in the extension of $T_i$ in some expansions of $\mathfrak{N}_{\mathscr{O}}$, but not in others. More precisely, a $k$-place predicate $T_i$ tripartitions $|\mathfrak{N}_{\mathscr{O}}|^k$ into a set $T_i^+$ of $k$-tuples of objects that are always in $T_i$'s extension (the *positive extension* of $T_i$), a set $T_i^-$ of $k$-tuples that are never in $T_i$'s extension (the *negative extension*), and a set of $k$-tuples that are only sometimes in $T_i$'s extension, which I will call $T^\circ$ (the *neutral extension*). In a generalization, Przełęcki (1976, 375)

[17] Incidentally, because of this psychological process of abstraction, Przełęcki (1974b, 403–404) contends that ostension is less restricted than Tuomela (1972, §2) makes it out to be in his criticism of Przełęcki. As to Tuomela's rejoinder that he "attributed to Przełęcki somewhat too strict a view on ostension, if [Przełęcki's] reply really states what he said in his monograph" (Tuomela 1974, 407): See pages 36–37.

also discusses the denotation of a function symbol $F$ that is vague over $|\mathfrak{N}_{\mathcal{O}}|$, which does not assign a single element $b \in |\mathfrak{N}_{\mathcal{O}}|$ to a $k$-tuple $\langle a_1, \ldots, a_k \rangle \in |\mathfrak{N}_{\mathcal{O}}|^k$, but rather a set $F^{+\circ}(a_1, \ldots, a_k) = B \subseteq |\mathfrak{N}_{\mathcal{O}}|$ (call this the '*non-negative extension* of $F$').[18] $B$ can be seen as the set of possible values of the function named by $F$ for the arguments $a_1, \ldots, a_k$. The denotation of a constant symbol $c$ (considered as a 0-place function symbol) that is vague over $|\mathfrak{N}_{\mathcal{O}}|$ is thus a set $c^{+\circ} \subseteq |\mathfrak{N}_{\mathcal{O}}|$.

In a keen move, Przełęcki (1969, ch. 5) draws on the multiplicity of possible interpretations to formally describe the semantics of vague *observational* terms. Because the interpretation of any $\mathcal{O}$-term is ostensive and some objects will be neither like its positive nor like its negative standards, these objects will be in the term's neutral extension (Przełęcki 1969, 38–39). Like Carnap, Przełęcki (1969, 34) distinguishes between the observable objects $O$ and the unobservable objects $U$ in the domain $O \cup U$ of $\Theta$. Unlike Carnap, Przełęcki (1969, 38) suggests the following delineation of observable objects:

> We shall call an object *observable*, if the possibility of its being observed is guaranteed by some natural law. In other words, $x$ is observable if $x$ has a property $P$ such that the following statement: whoever (in suitable conditions) looks at an object possessing property $P$ will perceive the object—is a statement of a natural law. This loose explication is not meant to serve as a definition of observability. It is only intended to point out some of its characteristic features.

Whether an object is observable or not is thus an empirical question. This does not mean that $\Theta$ itself determines the observable objects, since $\Theta$ can be one theory among many. The theories containing the natural laws about observability may be different from $\Theta$.

Since an unobservable object is similar to neither positive nor negative standards of any $\mathcal{O}$-term, Przełęcki (1969, 40–41) argues that all $\mathcal{O}$-terms are completely vague over $U$, that is, every object in $U$ is in the neutral or non-negative extension of every $\mathcal{O}$-term. This is markedly different from Carnap's approach, in which unobservable objects have to be in the negative extension of all $\mathcal{O}$-terms simply because unobservable objects are numbers. Przełęcki (1969, 40–41), on the other hand, intends unobservable objects to be physical (cf. Przełęcki 1974b, 405), which provides a second justification of his assumption of a fixed domain $O \cup U$. For a fixed domain of the theory blocks a proof by Winnie (1967), according to which there is for any unobservable object $a$ a model of $\Theta$ in which any other unobservable object (e. g. a number) is exchanged for $a$. Przełęcki thus chooses a fixed domain over the danger of an antirealist interpretation of unobservable objects. And this may be the biggest difference between Carnap's and Przełęcki's approaches: While Przełęcki aims to be a realist about unobservable objects, Carnap explicitly allows for an antirealist stance towards them.

[18]This is a slight generalization of Przełęcki's account, who assumes that $B$ is an interval of reals, which would therefore have to be in $|\mathfrak{N}_{\mathcal{O}}|$.

Incidentally, one can now also see a central restriction of Przełęcki's semantics in his monograph, which is explicitly acknowledged by Przełęcki (1969, 103–104): It does not contain mathematical objects, even though $\Theta$ could contain axiomatizations of mathematics. In subsequent discussions, however, Przełęcki (1974a, 347–348; 1976, 376) drops this restriction and allows terms of the theory to be directly and uniquely interpreted by mathematical objects. In other words, mathematical terms are interpreted like (non-vague) $\mathscr{O}$-terms. As Balzer and Reiter (1989) show, such a formalization is possible in a sorted first order language without the loss of completeness.

In Przełęcki's semantics the existence of unobservable objects, and in general the direct interpretation of $\mathscr{O}$-terms by ostension leads to a non-trivial class $\mathbf{N}_{\mathscr{O}}$ of intended $\mathscr{O}$-structures, and *these* then determine, together with $\mathrm{An}(\Theta)$, the class

$$\mathbf{N} := \{\mathfrak{N} : \mathfrak{N}|_{\mathscr{O}} \in \mathbf{N}_{\mathscr{O}} \wedge \mathfrak{N} \models \mathrm{An}(\Theta)\} \tag{4}$$

of intended $\mathscr{V}$-structures. The effect of this is that once $\mathbf{N}$ is determined, $\mathscr{O}$-terms and $\mathscr{T}$-terms can be treated in the same way (as interpreted non-uniquely by classes of intended structures), even though $\mathscr{O}$-terms are directly interpreted, and $\mathscr{T}$-terms are indirectly interpreted.

After a cogent discussion of different possibilities for defining truth and falsity in sets of structures (rather than single structures), Przełęcki (1969, 22) opts for conditional definitions: $\sigma$ is true in $\mathbf{N}$ if $\mathfrak{N} \models \sigma$ for all $\mathfrak{N} \in \mathbf{N}$, and $\sigma$ is false in $\mathbf{N}$ if $\mathfrak{N} \not\models \sigma$ for all $\mathfrak{N} \in \mathbf{N}$. As Przełęcki (1976, 375, n. 3) points out, the conditionals together with their converses make truth and falsity into *supertruth* and *superfalsity*, respectively (cf. Fine 1975), which I will assume for ease of exposition. Przełęcki (1976, 378–379) also suggests that the *subtruth* of $\sigma$ (cf. Hyde 1997), with $\mathfrak{N} \models \sigma$ for *some* $\mathfrak{N} \in \mathbf{N}$, can serve as an explication of the notion of *approximate truth*. Przełęcki (1976, 379, my notation) states:

> The notion of approximate truth makes it possible to dispense with certain idealizing assumptions in the semantics of empirical theories. The requirement that a physical theory $\Theta$ be approximately true allows to treat theory $\Theta$ as a theory of real, not idealized, objects. The universe of the structures in $\mathbf{N}$ may be thought of as a set of real things which are close enough to the alleged ideal entities. Thus, in the case of particle mechanics, its universe will be composed not of ideal point-masses, but of actual things, such as planets, projectiles, and the like. The theory is approximately true of them—in the sense being here considered. That is to say, among proper structures in $\mathbf{N}$ there [are] some in which the theory is (strictly) true. This amounts to the fact that the values of the theory's functions for those objects, as stated by the theory, fall into the intervals determined by the relevant measurement procedures.

For example, when measuring a magnitude $F$ with imperfect precision, an object $b$ (e. g., a body of condensed matter, liquid, or gas, or a whole system thereof) is assigned a range of possible values, and the pair consisting of $b$ and its range of values is in the non-negative extension $F^{+\circ}$ of $F$. The measurements and observations for all objects in the theory's domain and all relations and functions in $\mathscr{O}$ then determine, together with the domain itself, the class $\mathbf{N}_{\mathscr{O}}$ of intended $\mathscr{O}$-structures. If $\Theta$ contains only $\mathscr{O}$-terms, $\Theta$ is approximately true if and only if it is subtrue in $\mathbf{N}_{\mathscr{O}}$, and thus, because of its definition (4), also subtrue in $\mathfrak{N}$.

In general, of course, the definition of approximate truth in $\mathbf{N}$ hinges on the as-of-yet undefined notion of An($\Theta$). But under the assumption that $\Theta \vDash \mathrm{An}(\Theta)$ (discussed below), it is trivial to show that $\Theta$ is approximately true if and only if there is some $\mathfrak{N}_{\mathscr{O}} \in \mathbf{N}_{\mathscr{O}}$ that can be expanded to a model of $\Theta$. Delineating the analytic component of $\Theta$ is therefore not necessary for determining whether $\Theta$ is approximately true. But it is necessary for determining whether $\Theta$ is true in a vague language. For in that case, $\Theta$ has to be true in *every* $\mathfrak{N} \in \mathbf{N}$, and there may be intended structures that are not models of $\Theta$ since typically $\mathrm{An}(\Theta) \nvDash \Theta$.

## 3.2 The analytic-synthetic distinction

Towards a delineation of An($\Theta$), Przełęcki (1969, 58–59) assumes that $\Theta$ contains a distinguished set of postulates $P$ that determine the interpretation of the theory's theoretical terms. Since $P$ can have empirical content, Przełęcki allows $P \,\rightleftharpoons\, \Theta$ and I will assume this in the sequel. Przełęcki (1969, 50–51) then argues as follows:

> The language of any empirical theory always seems to be treated by the scientist as an interpreted, meaningful language, and not as a mere formal, meaningless calculus. And it seems to be treated so independently of any empirical findings. Experience may decide only whether an empirical theory is true or false, not whether it is meaningful or meaningless.

Since, first, an interpretation of the $\mathscr{T}$-terms is given by an expansion of some intended structure in $\mathbf{N}_{\mathscr{O}}$ to a model of An($\Theta$), and second, $\mathbf{N}_{\mathscr{O}}$ has to be determined exclusively by empirical means, Przełęcki (1969, 55, my notation) can conclude that An($\Theta$) must be "sufficiently weak to fulfil the semantic condition of non-creativity".

**Definition 4.** An($\Theta$) is *semantically non-creative* if and only if

$$\forall \mathfrak{A}_{\mathscr{O}} \exists \mathfrak{B} \,.\, \mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}_{\mathscr{O}} \wedge \mathfrak{B} \vDash \mathrm{An}(\Theta)\,. \tag{5}$$

*Any* $\mathscr{O}$-structure can be expanded to a model of An($\Theta$) if it is semantically non-creative. Thus An($\Theta$) is weak enough that it does not restrict the $\mathscr{O}$-structures.

"On the other hand", Przełęcki (1969, 55) argues, "it must be sufficiently strong to include all of the meaning postulates contained" in $\Theta$ . To express this demand formally, Przełęcki paraphrases a demand by Wójcicki (1963):[19]

**Definition 5.** An($\Theta$) *includes all of the meaning postulates* contained in $\Theta$ if and only if

$$\forall \mathfrak{A} \,.\, \exists \mathfrak{B} [\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}|_{\mathcal{O}} \wedge \mathfrak{B} \models \Theta] \rightarrow [\mathfrak{A} \models \mathrm{An}(\Theta) \leftrightarrow \mathfrak{A} \models \Theta] \,. \tag{6}$$

The conditions of adequacy for the analytic component of $\Theta$ are therefore given by

**Definition 6.** An($\Theta$) is a *semantically adequate analytic component* of $\Theta$ if and only if it is semantically non-creative and includes all of the meaning postulates contained in $\Theta$.

Incidentally, Przełęcki does not assume (and does not need to assume) that $\Theta$, An($\Theta$), and Syn($\Theta$) are single sentences. Hence I will treat them as sets in the following.

With Przełęcki's conditions of adequacy, one can now establish the already announced

**Claim 4.** *If* An($\Theta$) *is a semantically adequate analytic component of* $\Theta$*, then* $\Theta$ *is true (false) in the intended structures* $\mathbf{N}$ *if and only if every (no) intended* $\mathcal{O}$*-structure can be expanded to a model of* $\Theta$*.*

*Proof.* '$\Rightarrow$': Assume that $\Theta$ is true in $\mathbf{N}$. Then $\Theta$ is true in every $\mathfrak{N} \in \mathbf{N}$ and thus in every expansion of every intended $\mathcal{O}$-structure to a model of An($\Theta$). Since An($\Theta$) is semantically non-creative, $\Theta$ is thus true in at least one such expansion and, since An($\Theta$) contains all meaning postulates of $\Theta$, in all of them.

Now assume that some intended $\mathcal{O}$-structure can be expanded to a model of $\Theta$. Then it can be expanded to a model of An($\Theta$) because An($\Theta$) contains all meaning postulates of $\Theta$. Thus $\Theta$ is not false in $\mathbf{N}$.

'$\Leftarrow$': Assume that *every* $\mathfrak{N}_{\mathcal{O}} \in \mathbf{N}_{\mathcal{O}}$ can be expanded to a model of $\Theta$. Therefore, since An($\Theta$) includes all of the meaning postulates contained in $\Theta$, an expansion of $\mathfrak{N}_{\mathcal{O}}$ is a model of An($\Theta$) only if it is a model of $\Theta$. Hence every $\mathfrak{N} \in \mathbf{N}$ is a model of $\Theta$.

Now assume that *no* $\mathfrak{N}_{\mathcal{O}} \in \mathbf{N}_{\mathcal{O}}$ can be expanded to a model $\mathfrak{N}$ of $\Theta$. Since An($\Theta$) is semantically non-creative, no $\mathfrak{N} \in \mathbf{N}$ is a model of $\Theta$. □

Even without having determined a theory's analytic component, Przełęcki's semantics already provides the notion of approximate truth under the reasonable assumption that a theory entails its analytic component (which follows from claim 5 below). Claim 4 now shows that for truth in a vague language it is not necessary to determine a theory's analytic component either. Hence, while it is necessary to know how to delineate an analytic component of $\Theta$ in principle (by way of its conditions of adequacy), it is *not* necessary to do so in any specific case.

[19] I am very grateful to Krystian Jobczyk for giving me an overview and partial translation of Wójcicki's article.

Also based on work by Wójcicki (1963), Przełęcki and Wójcicki (1969, 376) give conditions of adequacy for the synthetic component of $\Theta$, based on

**Definition 7.** An $\mathscr{O}$-structure $\mathfrak{A}_{\mathscr{O}}$ is *admitted* by $\Theta$ if and only if

$$\exists\mathfrak{B}\,.\,\mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}_{\mathscr{O}} \wedge \mathfrak{B} \vDash \Theta\,. \tag{7}$$

Two sets of sentences that admit the same $\mathscr{O}$-structures place the same restrictions on them, and thus can be considered semantically empirically equivalent:

**Definition 8.** $\mathrm{Syn}(\Theta)$ and $\Theta$ are *semantically empirically equivalent* if and only if they admit the same $\mathscr{O}$-structures.

Przełęcki and Wójcicki (1969, 387) demand that $\mathrm{Syn}(\Theta)$ admit the same $\mathscr{O}$-structures as $\Theta$, so $\mathrm{Syn}(\Theta)$ and $\Theta$ must be semantically empirically equivalent. They demand further that $\mathrm{Syn}(\Theta)$ provide no empirical interpretation of $\mathscr{T}$.

**Definition 9.** $\mathrm{Syn}(\Theta)$ *provides no empirical interpretation* of $\mathscr{T}$ if and only if

$$\forall\mathfrak{A}\forall\mathfrak{B}\,.\,\mathfrak{A}|_{\mathscr{O}} = \mathfrak{B}|_{\mathscr{O}} \rightarrow [\mathfrak{A} \vDash \mathrm{Syn}(\Theta) \leftrightarrow \mathfrak{B} \vDash \mathrm{Syn}(\Theta)] \tag{8}$$

Together, these demands lead to

**Definition 10.** $\mathrm{Syn}(\Theta)$ is a *semantically adequate synthetic component* of $\Theta$ if and only if $\mathrm{Syn}(\Theta)$ is semantically empirically equivalent to $\Theta$ and provides no empirical interpretation of $\mathscr{T}$.

The conditions of adequacy for $\mathrm{An}(\Theta)$ and $\mathrm{Syn}(\Theta)$ can be combined as follows:

**Claim 5.** $\mathrm{An}(\Theta)$ *is a semantically adequate analytic component of* $\Theta$ *and* $\mathrm{Syn}(\Theta)$ *is a semantically adequate synthetic component of* $\Theta$ *if and only if*

1. $\mathrm{An}(\Theta) \cup \mathrm{Syn}(\Theta) \mathrel{\vDash\!\!\dashv} \Theta$,
2. $\mathrm{Syn}(\Theta)$ *is semantically empirically equivalent to* $\Theta$.
3. $\mathrm{Syn}(\Theta)$ *provides no empirical interpretation of* $\mathscr{T}$*, and*
4. $\mathrm{An}(\Theta)$ *is semantically non-creative.*

*Proof.* '$\Rightarrow$': Przełęcki and Wójcicki (1969, theorem 7) note that condition 1 follows from the conditions of adequacy; that the others follow is trivial.

'$\Leftarrow$': $\mathrm{Syn}(\Theta)$ is trivially a semantically adequate synthetic component of $\Theta$, and $\mathrm{An}(\Theta)$ is trivially semantically conservative. It remains to be shown that $\mathrm{An}(\Theta)$ contains all of the meaning postulates in $\Theta$. This is the case if and only if

$$\forall\mathfrak{A}\,.\,\exists\mathfrak{B}[\mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}|_{\mathscr{O}} \wedge \mathfrak{B} \vDash \Theta] \wedge \mathfrak{A} \vDash \Theta \rightarrow \mathfrak{A} \vDash \mathrm{An}(\Theta)\,, \tag{9}$$

and

$$\forall \mathfrak{A}\,.\,\exists \mathfrak{B}[\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}|_{\mathcal{O}} \wedge \mathfrak{B} \vDash \Theta] \wedge \mathfrak{A} \vDash \mathrm{An}(\Theta) \rightarrow \mathfrak{A} \vDash \Theta\,. \tag{10}$$

Conditional (9) is equivalent to $\Theta \vDash \mathrm{An}(\Theta)$ and follows trivially from condition 1. Conditional (10) follows from conditions 1, 2, and 3: Because of the first conjunct of the antecedent and condition 2, $\exists \mathfrak{B}[\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}|_{\mathcal{O}} \wedge \mathfrak{B} \vDash \mathrm{Syn}(\Theta)]$. Hence, by condition 3, $\mathfrak{A} \vDash \mathrm{Syn}(\Theta)$. $\mathfrak{A} \vDash \Theta$ then follows from the second conjunct of the antecedent and condition 1. □

Although the conditions of adequacy in claim 5 are arguably more intuitive than those given by Przełęcki and Wójcicki, their conditions have an indisputable advantage: Unlike claim 5, Przełęcki and Wójcicki's conditions can be applied to a purported analytic component without knowing the synthetic component and vice versa. This will be convenient in §4.3 below.

The conditions of claim 5 seem very similar to Carnap's (definition 3), and indeed, they are direct generalizations of Carnap's conditions to sets of sentences. The connection is given by the paraphrase of Carnap's conditions in terms of Ramsey sentences (claim 2) and

**Lemma 6.** *$\mathfrak{A}_{\mathcal{O}}$ can be expanded to a model of the sentence $\Phi$ if and only if $\mathfrak{A}_{\mathcal{O}} \vDash \mathsf{R}_{\mathcal{O}}(\Phi)$.*

*Proof.* Let $\Phi^\dagger$ be the result of substituting each $\mathcal{T}$-term in $\Phi$ by a corresponding variable.

'$\Leftarrow$': Since $\mathfrak{A}_{\mathcal{O}} \vDash \mathsf{R}_{\mathcal{O}}(\Phi)$, there is a relation $V_i$ for every relation symbol $P_i$ in $\mathcal{T}$, a function $G_j$ for every function symbol $F_j$ in $\mathcal{T}$, and a constant $d_k$ for every constant symbol $c_k$ in $\mathcal{T}$ such that $\{V_i, G_j, d_k\}$ satisfies $\Phi^\dagger$ in $\mathfrak{A}$. Define $\mathfrak{C}$ so that $P_i^{\mathfrak{C}} = V_i$ for each $V_i$, $F_j^{\mathfrak{C}} = G_j$ for each $G_j$, $c_k^{\mathfrak{C}} = d_k$ for every $d_k$, and $\mathfrak{C}|_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}}$. Induction on the complexity of $\Phi$ shows that $\mathfrak{C} \vDash \Phi$.

'$\Rightarrow$': Induction shows that $\{P_i^{\mathfrak{C}}, F_j^{\mathfrak{C}}, c_k^{\mathfrak{C}}\}$ satisfies $\Phi^\dagger$ in $\mathfrak{A}$, so $\mathfrak{A} \vDash \exists_i X_i \exists_j Y_j \exists_k x_k \Phi^\dagger$. □

Lemma 6 immediately entails

**Corollary 7.** *Let $\Theta$, $\mathrm{Syn}(\Theta)$, and $\mathrm{An}(\Theta)$ be single sentences. Then $\mathrm{Syn}(\Theta)$ is semantically equivalent to $\Theta$ if and only if $\mathsf{R}_{\mathcal{O}}(\mathrm{Syn}(\Theta)) \vDash\!\dashv \mathsf{R}_{\mathcal{O}}(\Theta)$ and $\mathrm{An}(\Theta)$ is semantically non-creative if and only if $\vDash \mathsf{R}_{\mathcal{O}}(\mathrm{An}(\Theta))$.*

Furthermore the following holds:

**Lemma 8.** *Let $\mathrm{Syn}(\Theta)$ be a single sentence. Then $\mathrm{Syn}(\Theta)$ provides no empirical interpretation of $\mathcal{T}$ if and only if $\mathrm{Syn}(\Theta) \vDash\!\dashv \mathsf{R}_{\mathcal{O}}(\mathrm{Syn}(\Theta))$.*

*Proof.* '$\Rightarrow$: Since $\mathsf{R}_{\mathcal{O}}(\mathrm{Syn}(\Theta))$ is an existential generalization of $\mathrm{Syn}(\Theta)$, it is clear that $\mathrm{Syn}(\Theta) \vDash \mathsf{R}_{\mathcal{O}}(\mathrm{Syn}(\Theta))$. For the converse, assume that $\mathfrak{A} \vDash \mathsf{R}_{\mathcal{O}}(\mathrm{Syn}(\Theta))$. By lemma 6, there is a $\mathfrak{B} \vDash \mathrm{Syn}(\Theta)$ with $\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}|_{\mathcal{O}}$. Therefore $\mathfrak{A} \vDash \Theta$.

'$\Leftarrow$': Immediate. □

Corollary 7 and lemma 8 furthermore entail

**Claim 9.** *Let* $\Theta$ *and* $\mathrm{Syn}(\Theta)$ *be single sentences. Then* $\mathrm{Syn}(\Theta)$ *is a semantically adequate synthetic component of* $\Theta$ *if and only if* $\mathrm{Syn}(\Theta) \boxminus \mathsf{R}_{\mathscr{O}}(\Theta)$.

*Proof.* If $\mathrm{Syn}(\Theta)$ is a semantically adequate synthetic component of $\Theta$, then $\mathsf{R}_{\mathscr{O}}(\Theta) \boxminus \mathsf{R}_{\mathscr{O}}(\mathrm{Syn}(\Theta))$ and $\mathsf{R}_{\mathscr{O}}(\mathrm{Syn}(\Theta)) \boxminus \mathrm{Syn}(\Theta)$, and thus $\mathrm{Syn}(\Theta) \boxminus \mathsf{R}_{\mathscr{O}}(\Theta)$. The converse holds because $\mathsf{R}_{\mathscr{O}}(\mathsf{R}_{\mathscr{O}}(\Theta)) \boxminus \mathsf{R}_{\mathscr{O}}(\Theta)$. □

Because of claim 9, a semantically adequate synthetic component of $\Theta$ must be equivalent to a (singleton) set of $\mathscr{O}$-sentences. Hence claim 2, corollary 7, corollary 8, and claim 9 together entail

**Claim 10.** *Let* $\Theta$, $\mathrm{An}(\Theta)$, *and* $\mathrm{Syn}(\Theta)$ *be single sentences. Then* $\mathrm{An}(\Theta)$ *is a semantically adequate analytic component of* $\Theta$ *and* $\mathrm{Syn}(\Theta)$ *is a semantically adequate synthetic component of* $\Theta$ *if and only if* $\mathrm{An}(\Theta)$ *is an adequate analytic component of* $\Theta$ *and* $\mathrm{Syn}(\Theta)$ *is, up to equivalent reformulation, an adequate synthetic component of* $\Theta$ *according to definition 3.*

Thus for single sentences, Wójcicki and Przełęcki's conditions of adequacy are essentially equivalent to Carnap's and can be phrased in a very compact way in terms of of Ramsey sentences.

Claim 9 shows that Przełęcki and Wójcicki's (and hence Carnap's) conditions of adequacy uniquely determine the synthetic component of $\Theta$ up to logical equivalence when $\mathscr{O}$-sentences can be of any order. In contradistinction, $\mathrm{An}(\Theta)$ is not in general uniquely determined by their conditions of adequacy, as Przełęcki and Wójcicki (1969, 391) point out. In particular, the Carnap sentence is just the weakest of a variety of adequate analytic components of $\Theta$. Przełęcki (1969, §7.III) provides a nice example for reduction sentences

$$\Theta \boxminus \forall x[\varphi(x) \to T_1 x] \wedge \forall x[\psi(x) \to \neg T_1 x]\,, \tag{11}$$

where $\varphi$ and $\psi$ are $\mathscr{O}$-formulas and $T_1$ is a $\mathscr{T}$-term. Then

$$\mathsf{C}_{\mathscr{O}}(\Theta) \boxminus \forall x[\varphi(x) \to \neg\psi(x)] \to \forall x[\varphi(x) \to T_1 x] \wedge \forall x[\psi(x) \to \neg T_1 x] \tag{12}$$

is an adequate analytic component of $\Theta$. However, the logically stronger sentence

$$\forall x[\varphi(x) \wedge \neg\psi(x) \to T_1 x] \wedge \forall x[\psi(x) \wedge \neg\varphi(x) \to \neg T_1 x] \tag{13}$$

is also an adequate analytic component of $\Theta$. This solution has general advantages (Przełęcki 1961b) and can, for example, be used to salvage ethical terms that were introduced by reduction sentences with empirically false implications (Lutz 2010). Winnie (1970, 294–296) and Demopoulos (2007, V) argue that the possibility of choosing the analytic component of a theory is a problem for the analytic-synthetic distinction, and they suggest an additional condition of adequacy that establishes a theory's Carnap sentence as its sole adequate analytic component. However, neither their argument nor their suggested condition of adequacy are convincing (Caulton 2012; Lutz 2012a, §12.1).

### 3.3 Przełęcki's modifications of his semantics

Przełęcki (1969, §10.I) generalizes his semantics in a significant way already in his monograph. He introduces a hierarchy of languages $\mathcal{O}, \mathcal{T}_1, \mathcal{T}_2, \ldots$ that begins with a set $\mathcal{O}$ of observational terms and continues with a series of sets of theoretical terms. The intended $\mathcal{O}$-structures and some theory $\Theta_1$ determine the intended structures for $\mathcal{O} \cup \mathcal{T}_1$, which in turn determines with some theory $\Theta_2$ the intended structures for $\mathcal{O} \cup \mathcal{T}_1 \cup \mathcal{T}_2$, and so forth. The intended $\mathcal{O}$-structures do not interpret the vocabulary of the theory $\Theta_n$, $n > 1$, so the class of intended structures for $\Theta_n$ is the reduct of the intended structures for $\mathcal{O} \cup \mathcal{T}_1 \cup \cdots \cup \mathcal{T}_n$ to $\mathcal{T}_{n-1} \cup \mathcal{T}_n$. Indeed, there need not even be a well-determined class of $\mathcal{O}$-terms; it may simply be convenient within the formalism to postulate this starting point of the hierarchy (Przełęcki 1969, §10.II). Such a hierarchy of vocabularies was suggested at about the same time by Rozeboom (1970, 202), and Przełęcki's formalism is a plausible explication of the hierarchies of vocabularies discussed by Carnap. For convenience, $\mathcal{T}_n$ and $\mathcal{T}_{n-1}$ can be renamed to $\mathcal{T}$ and $\mathcal{O}$, respectively, keeping in mind that $\mathcal{O}$ does not have to be observational in any specific sense. In fact, Carnap (1931, 437–438) suggested that observation reports are formulated more expediently in such a vocabulary.

The new $\mathcal{O}$-terms can thus be considered to refer to concepts that are not themselves under investigation, as was suggested explicitly by Reichenbach (1951, 49), Lewis (1970, 428), and Carnap himself. Since the $\mathcal{O}$-terms are unproblematic in this sense, there can be a set $\Pi_{\mathcal{O}}$ of analytic $\mathcal{O}$-sentences. Accordingly, Przełęcki (1969, 98–99) generalizes the conditions of adequacy for a theory's analytic component:

**Definition 11.** An($\Theta$) is a *semantically adequate analytic component of* $\Theta$ *given analytic* $\mathcal{O}$*-sentences* $\Pi_{\mathcal{O}}$ if and only if

$$\forall \mathfrak{A}_{\mathcal{O}} . \mathfrak{A}_{\mathcal{O}} \models \Pi_{\mathcal{O}} \to \exists \mathfrak{B} . \mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}} \wedge \mathfrak{B} \models \mathrm{An}(\Theta) \tag{14}$$

and

$$\forall \mathfrak{A} . \mathfrak{A} \models \Pi_{\mathcal{O}} \wedge \exists \mathfrak{B}[\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}|_{\mathcal{O}} \wedge \mathfrak{B} \models \Theta] \to [\mathfrak{A} \models \mathrm{An}(\Theta) \leftrightarrow \mathfrak{A} \models \Theta] . \tag{15}$$

The first condition (14) generalizes semantic non-creativity to semantic non-creativity relative to $\mathcal{O}$-sentences $\Pi_{\mathcal{O}}$[20] and the second condition generalizes the demand that An($\Theta$) include all of the meaning postulates of $\Theta$.

In a later work, Przełęcki (1973) discusses the possibilities for avoiding the assumption of a fixed domain. The reason is simple: Fixing the set $U$ of unobservable objects in the metalanguage is impossible if the interpretation of the $\mathcal{T}$-terms is to be determined solely by the interpretation of the $\mathcal{O}$-terms and by $\Theta$. Therefore the assumption of a fixed domain $O \cup U$ is incompatible with the

[20] This can be further generalized to semantic non-creativity relative to $\mathcal{V}$-sentences (Lutz 2012a, definition 6.6, cf. §6.11.1).

thesis of semantic empiricism (Przełęcki 1974b, 405). Przełęcki (1973, 287) avoids the assumption in two alternative modifications of his semantics. Given the intended $\mathscr{O}$-structure $\mathfrak{N}_{\mathscr{O}}$, the intended $\mathscr{V}$-structures can be given by the class of all expansions of elementary extensions of $\mathfrak{N}_{\mathscr{O}}$ or the class of all expansions of extensions of $\mathfrak{N}_{\mathscr{O}}$. If there is no single intended observational structure $\mathfrak{N}_{\mathscr{O}}$ but a class $\mathbf{N}_{\mathscr{O}}$ thereof, the class of intended structures may be defined in three different ways:

$$\mathbf{N} := \{\mathfrak{A} : \mathfrak{A} \vDash \mathrm{An}(\Theta) \wedge \exists \mathfrak{B}_{\mathscr{O}} \in \mathbf{N}_{\mathscr{O}} . \mathfrak{B}_{\mathscr{O}} = \mathfrak{A}|_{\mathscr{O}}\} \tag{16}$$

$$\mathbf{N} := \{\mathfrak{A} : \mathfrak{A} \vDash \mathrm{An}(\Theta) \wedge \exists \mathfrak{B}_{\mathscr{O}} \in \mathbf{N}_{\mathscr{O}} . \mathfrak{B}_{\mathscr{O}} \preceq \mathfrak{A}|_{\mathscr{O}}\} \tag{17}$$

$$\mathbf{N} := \{\mathfrak{A} : \mathfrak{A} \vDash \mathrm{An}(\Theta) \wedge \exists \mathfrak{B}_{\mathscr{O}} \in \mathbf{N}_{\mathscr{O}} . \mathfrak{B}_{\mathscr{O}} \subseteq \mathfrak{A}|_{\mathscr{O}}\} \tag{18}$$

The first definition (16) paraphrases Przełęcki's initial definition (4). The two modifications (17) and (18) avoid the assumption of a fixed domain and are compatible with the thesis of semantic empiricism, but are also subject to Winnie's proof: In both semantics, an intended $\mathscr{O}$-structure can always be extended to contain mathematical objects if it can be extended at all.

Przełęcki (1973, 289) modifies the condition of non-creativity accordingly, assuming the possibility of analytic $\mathscr{O}$-sentences. When elementary extensions of the intended $\mathscr{O}$-structures are allowed, this leads to

**Definition 12.** $\mathrm{An}(\Theta)$ is *up to elementary extensions semantically non-creative relative to* $\mathscr{O}$-sentences $\Pi_{\mathscr{O}}$ if and only if

$$\forall \mathfrak{A}_{\mathscr{O}} . \mathfrak{A}_{\mathscr{O}} \vDash \Pi_{\mathscr{O}} \rightarrow \exists \mathfrak{B} . \mathfrak{A}_{\mathscr{O}} \preceq \mathfrak{B}|_{\mathscr{O}} \wedge \mathfrak{B} \vDash \mathrm{An}(\Theta) . \tag{19}$$

As Przełęcki points out, definition 4 has no syntactic formulation in first order logic, while definition 12 corresponds to syntactic non-creativity:

**Definition 13.** $\mathrm{An}(\Theta)$ is *syntactically non-creative relative to* $\mathscr{O}$-sentences $\Pi_{\mathscr{O}}$ if and only if for all $\mathscr{O}$-sentences $\omega$, $\mathrm{An}(\Theta) \cup \Pi_{\mathscr{O}} \vDash \omega$ only if $\Pi_{\mathscr{O}} \vDash \omega$.

**Claim 11.** $\mathrm{An}(\Theta)$ *is up to elementary extensions semantically non-creative relative to $\mathscr{O}$-sentences* $\Pi_{\mathscr{O}}$ *if and only if* $\mathrm{An}(\Theta)$ *is syntactically non-creative relative to $\mathscr{O}$-sentences* $\Pi_{\mathscr{O}}$.

The difference between semantic and syntactic non-creativity lies at the core of the critique of Demopoulos and Friedman (1985) by Ketland (2004, 297–299).[21] Demopoulos's response turns on the equivalence of syntactic non-creativity and semantic non-creativity up to elementary extension (Demopoulos 2011, §4).

In the case where the semantics allows for any kind of extensions of the $\mathscr{O}$-structures, Przełęcki (1973, 289) modifies semantic non-creativity as follows:

**Definition 14.** $\mathrm{An}(\Theta)$ is *up to extensions semantically non-creative relative to* $\mathscr{O}$-sentences $\Pi_{\mathscr{O}}$ if and only if

$$\forall \mathfrak{A}_{\mathscr{O}} . \mathfrak{A}_{\mathscr{O}} \vDash \Pi_{\mathscr{O}} \rightarrow \exists \mathfrak{B} . \mathfrak{A}_{\mathscr{O}} \subseteq \mathfrak{B}|_{\mathscr{O}} \wedge \mathfrak{B} \vDash \mathrm{An}(\Theta) . \tag{20}$$

[21] As Przełęcki and Wójcicki (1971, 94–95) show, this difference holds even if $\Pi_{\mathscr{O}} = \varnothing$.

Przełęcki (1973, 289) notes

**Claim 12.** $\mathrm{An}(\Theta)$ *is up to extensions semantically non-creative relative to $\mathcal{O}$-sentences $\Pi_{\mathcal{O}}$ if and only if for all purely universally $\mathcal{O}$-sentences $\omega$,* $\mathrm{An}(\Theta) \cup \Pi_{\mathcal{O}} \vDash \omega$ *only if* $\Pi_{\mathcal{O}} \vDash \omega$.

Note that in the semantics that allows any kind of extensions of intended $\mathcal{O}$-structures, the truth-value of a first order $\mathcal{O}$-sentence can be different in $\mathbf{N}_{\mathcal{O}}$ and $\mathbf{N}$, which is impossible in the previous two.

It is clear that Przełęcki's modifications of his semantics lead to natural modifications of definition 6 for an adequate analytic component and of definition 10 for an adequate synthetic component of a theory. Przełęcki demonstrates this with his modifications of non-creativity (definitions 12 and 14); the general method can be read off of these: To take previously accepted analytic sentences $\Pi_{\mathcal{O}}$ into account, all that is needed is a restriction of the quantifiers over the class of $\mathcal{O}$ structures to the class of $\mathcal{O}$-models of $\Pi_{\mathcal{O}}$. To take the change from simple expansions to expansions of elementary extensions or expansions of extensions into account, all that is needed is a systematic substitution of '$=$' between structures by '$\preceq$' or '$\subseteq$', respectively.

## 4 After the Received View

In the preceding section, I have argued that Przełęcki's semantics can be seen as an explication of Carnap's informal description of the semantics of scientific theories. I now want to argue that with his semantics, Przełęcki both influenced and anticipated later developments in the philosophy of science. Although Przełęcki (1975, 284) considered himself "positivistically-minded" and his monograph an introduction to the Received View, he had the greatest influence on the Received View's main opponent—the so-called Semantic View—which describes theories as classes of model theoretic structures rather than sets of sentences. Probably due to Przełęcki's heavy use of model theoretic methods, da Costa and French (1990, 249) consider his monograph a precursor of the Semantic View and Volpe (1995, 566) even lists it as an early work *within* the Semantic View.[22] It is notable that, even though Przełęcki's monograph does not seem to have had any specific influence on the article by da Costa and French (1990), Przełęcki (1969; 1976) anticipates and even generalizes their central concepts of partial structures and quasi-truth in his concepts of vague terms and approximate truth (Lutz 2012a, §4.3.2).

[22]This suggests that the differences between the Received View and the Semantic View are not as large as some proponents of the latter have claimed (cf. Lutz 2012a, 4.1). Przełęcki (1974c) also outlines how to bridge the gap to Sneed's Structuralist View, which is sometimes considered one variety of the Semantic View.

### 4.1 Constructive empiricism

Przełęcki explicitly influenced van Fraassen, who famously declared that the Received View is in principle unable to capture the relation between theory and phenomena (van Fraassen 1980, §3.6). Within his alternative, constructive empiricism, van Fraassen (1980, 64) states that

> [t]o present a theory is to specify a family of structures, its *models*; and secondly, to specify certain parts of those models (the *empirical substructures*) as candidates for the direct representation of observable phenomena.

Furthermore the models of the theory "are describable only up to structural isomorphism" (van Fraassen 2008, 238; cf. 2002, 22). Under the simplifying assumption that each model of the theory has exactly one empirical substructure, this can be phrased as follows:

**Definition 15.** A *theory* is a family $\{\mathfrak{T}_n\}_{n \in N}$ of structures (the *models of the theory*) such that each of its members $\mathfrak{T}_n$ has exactly one *empirical substructure* $\mathfrak{E}_n \subseteq \mathfrak{T}_n$. With each model, a theory also contains every isomorphic structure with its corresponding empirical substructure.

Van Fraassen (1980, 64) strictly distinguishes between the set $O$ of observable objects and the unobservable objects and suggests describing observable phenomena by structures as well: "The structures which can be described in experimental and measurement reports we can call *appearances*" (van Fraassen 2008, 286). With the simplifying assumption that all appearances can be included in a single structure, this suggests

**Definition 16.** The *appearances* are given by a structure $\mathfrak{P}$ with $|\mathfrak{P}| = O$.

Van Fraassen (1980, 64) then defines a theory as "empirically adequate if it has some model such that all appearances are isomorphic to empirical substructures of that model" (cf. van Fraassen 1991, 12). With the simplifications given above, this results in

**Definition 17.** A theory $\{\mathfrak{T}_n\}_{n \in N}$ is *empirically adequate* for the appearance $\mathfrak{P}$ if and only if there is some $n \in N$ such that $\mathfrak{E}_n \cong \mathfrak{P}$.

Van Fraassen (1980, 64; 1989, 227) traces his inspiration for relying on empirical substructures to Przełęcki's monograph, but a much more direct connection is given by Przełęcki's second modification of his semantics:

**Claim 13.** *Assume that* $\{\mathfrak{T}_n : n \in N\} = \{\mathfrak{B} : \mathfrak{B} \vDash \Theta\}$, *that* $\Theta$ *contains only* $\mathcal{O}$-*terms, and that all and only expansions of extensions of intended* $\mathcal{O}$-*structures are in* $\mathbf{N}$. *Then, if* $\mathbf{N}_{\mathcal{O}} = \{\mathfrak{P}\}$, $\{\mathfrak{T}_n\}_{n \in N}$ *is empirically adequate if and only if* $\Theta$ *is approximately true in* $\mathbf{N}$.

*Proof.* $\{\mathfrak{T}_n\}_{n\in N}$ is empirically adequate for $\mathfrak{P}$ if and only if $\mathfrak{P}$ has an extension to some $\mathfrak{T}_n, n \in N$. This holds if and only if $\mathfrak{T}_n \in \mathbf{N}$, that is, $\mathfrak{N} \models \Theta$ for some $\mathfrak{N} \in \mathbf{N}$, which is the approximate truth of $\Theta$. □

The first assumption in claim 13 is a restriction on van Fraassen's notion of a theory, because the language of $\Theta$ may not be strong enough to describe $\{\mathfrak{T}_n\}_{n\in N}$ up to isomorphism. The second assumption is a restriction on Przełęcki's notion of a theory, which allows theories to contain terms that do not occur in the description of the appearances. The third assumption simply describes Przełęcki's second modification of his semantics. The condition $\mathbf{N}_{\mathcal{O}} = \{\mathfrak{P}\}$ shows very clearly that within the domain of observable objects, van Fraassen assumes perfect information about the phenomena without any vagueness of terms or imprecision of measurement. Van Fraassen (1989, 366, n. 5) is aware of this, as he suggests introducing an approximate notion of empirical substructures to generalize empirical adequacy. In Przełęcki's terminology, and in full agreement with Przełęcki's own position, the $\mathcal{O}$-terms are completely vague for the unobservable objects.

Together with claim 12, claim 13 has the nice

**Corollary 14.** *Assume a language of first order logic. Assume further that $\{\mathfrak{T}_n : n \in N\} = \{\mathfrak{B} : \mathfrak{B} \models \Theta\}$, that $\Theta$ contains only $\mathcal{O}$-terms, and that all and only expansions of extensions of intended $\mathcal{O}$-structures are in* $\mathbf{N}$. *Then if $\mathfrak{P}$ is described up to isomorphism by $\Omega$, $\{\mathfrak{T}_n\}_{n\in N}$ is empirically adequate if and only if $\Theta$ is compatible with all purely universal $\mathcal{O}$-sentences entailed by $\Omega$.*

*Proof.* Since $\Omega$ describes $\mathfrak{P}$ up to isomorphism, $\Theta$ is approximately true in $\mathbf{N}$ if and only if $\Theta$ is up to extensions semantically non-creative relative to $\Omega$, which is the case if and only if for every purely universal $\mathcal{O}$-sentence $\omega$, $\Theta \cup \Omega \models \omega$ only if $\Omega \models \omega$. Since $\Omega$ is maximally consistent, this holds if and only if $\Theta$ is compatible with all purely universal $\mathcal{O}$-sentences entailed by $\Omega$. □

An analogue to corollary 14 for another semantic concept of van Fraassen's was later pointed out by Scott Weinstein in a defense of "syntactic approaches" like the Received View (Friedman 1982, 277) . This would be the second time that Przełęcki anticipated a defense of the Received View.

### 4.2 Suppe on partial interpretations

A defense by Suppe (1971) was possibly the first that Przełęcki anticipated. Although highly critical of the Received View (Suppe 1972), Suppe gives an elucidation of the Received View's notion of a 'partial interpretation' with the aim of defending it against Putnam's claim that the notion was "completely broken-backed" (Putnam 1962, 241). Towards this goal, Suppe develops a number of supporting concepts. First, he assumes that the language of the theory $\Theta$ is of first order and that the $\mathcal{O}$-terms are interpreted over a domain of "concrete observable entities" (Suppe 1972, 58–59, my notation here and in the following). Suppe (1972,

60) goes on to "assume that a fixed set of rules of designation has been specified" for the $\mathscr{O}$-terms and calls the class of interpretations specified by these rules the *permissible interpretations* $S^*$ for $\mathscr{O}$. The "result of adding a new property $P$ to $[S^*]$ and a new rule of designation that 'P' designates $P$ is said to be a *permissible extension* of $[S^*]$" (Suppe 1971, 63). Suppe (1971, 65) adds that for a permissible extension from $S$ to $S^*$, "we do not require that $S$ and $S^*$ have the same domain, but rather only that the domain of $S$ contain the domain of $S^*$. This allows the possibility that the domain of $S$ may contain both theoretical entities and observable entities, etc".[23] Finally, in effect treating the correspondence rules $C$ (from $TC = T \cup C \boxminus \Theta$) like the postulates that determine the interpretation of $\mathscr{T}$ in Przełęcki's account,[24] Suppe (1971, 65) concludes that

> the assumed truth of $TC$ will impose restrictions upon the class of true permissible extensions to $[\mathscr{T}]$. [...] This, then, suggests that the sense in which the interpretative system $C$ supplies $[\mathscr{T}]$ with a partial interpretation is that it imposes restrictions on the class of permissible models for it.

The resulting semantics is that of Przełęcki's second modification: The $\mathscr{O}$-terms are interpreted by an $\mathscr{O}$-structure $S$, and an expansion $S^*$ of an extension is a "permissible extension" in Suppe's sense if and only if $S \subseteq S^*|_{\mathscr{O}}$.[25]

Przełęcki (1973) suggested his second modification at a conference in 1971 (Bogdan and Niiniluoto 1973, v), in the year that Suppe (1971) published his semantics. But it seems that the core idea of Suppe's semantics is that of letting the interpretation of $\mathscr{O}$ fix a structure $\mathfrak{N}_{\mathscr{O}}$ and consider any structure a possible interpretation of $\mathscr{V}$ that in some sense includes $\mathfrak{N}_{\mathscr{O}}$; and this account is already worked out with much precision and in a very general way in Przełęcki's monograph. Of course, the question of priority is basically moot since Suppe and Przełęcki have developed their accounts wholly independently. The important point is rather that their accounts are equivalent, for this provides another reason to consider Przełęcki's semantics a faithful formal account of the semantics of the Received View, since such an account was an explicit goal of Suppe's.

Unfortunately, Suppe (1971, 67) further argues that within the constraints of the Received View, his semantics can be supplemented by a direct interpretation of the $\mathscr{T}$-terms within some antecedently understood metalanguage, which he assumes to be a natural language. But this goes against the thesis of semantic empiricism and makes partial interpretation pointless, since there is no need to

[23] Suppe's sequence of definitions takes a number of twists and turns that are sometimes difficult to follow, so that the relation between $S^*$ and the permissible extensions $S$ might be not as direct as indicated here. This may be partly because of his puzzling use of some technical terms. For example Suppe (1971, 59, 60) repeatedly speaks of predicate and function variables although the language is of first order. I hope to have done his semantics justice.

[24] As noted, I have instead used the whole of $\Theta$.

[25] As noted above, Suppe's outline of his semantics is not always clear, and I must restrict my claim to the extent that I have been able to reconstruct Suppe's semantics.

construct an interpretation of the $\mathscr{T}$-terms with the help of $\Theta$ if the $\mathscr{T}$-terms are already directly interpreted.[26] Furthermore, Suppe's move only pushes questions about the interpretation of theoretical terms of the object language into the metalanguage, as was already argued by Carnap (1939, 204), Hempel (1963, 696), and Rozeboom (1970, 204–205), and again pointed out by Przełęcki (1974b, 402) in a response to a similar suggestion by Tuomela (1972, 171). More generally, Suppe's (and Tuomela's) suggestion seems to stem from "a certain tendency of ascribing to natural language a 'mystical' quality not inherent in formalized languages". In this way Beth (1963, 481) describes the view that natural languages can provide intended interpretations in a way that formalized languages cannot. Without such a mystical view, supplementing a formal theory $\Theta$ with natural language sentences "is just adding more theory", as Putnam (1980, 477, emphasis removed) put it most famously.

### 4.3 Andreas on the semantics of scientific theories

In a more recent article, Andreas (2010, 530–532) aims to give a formal semantics based on the indirect interpretation of theoretical terms outlined by Carnap (1939). Like Carnap and Przełęcki, Andreas assumes a fixed domain. And like Carnap, but unlike Przełęcki, he assumes that there is a fixed bipartition of the domain into observable objects $O$ and mathematical objects $U$. Andreas (2010, 529, 532) further introduces the notion of an intended interpretation, specifically the intended interpretation $\mathfrak{N}_{\mathscr{O}}$ of the $\mathscr{O}$-terms; like Przełęcki and Suppe, he assumes that there is a subset $P$ of postulates of $\Theta$ that determine the interpretation of the theory's theoretical terms. Also like Przełęcki, Andreas assumes that it is possible and indeed preferable to let $P = \Theta$,[27] which I will suppose in the following. With these concepts, Andreas (2010, 533) suggests that the intended interpretations of $\mathscr{V}$ be given by the admissible structures for the intended $\mathscr{O}$-structure $\mathfrak{N}_{\mathscr{O}}$ and the theory $\Theta$.

**Definition 18.** Let $O \cap U = \varnothing$ and let $\mathfrak{N}_{\mathscr{O}}$ be the intended $\mathscr{O}$-structure with $|\mathfrak{N}_{\mathscr{O}}| = O$. Define two classes of $\mathscr{V}$-structures:

$$\mathrm{S}_1(\mathfrak{N}_{\mathscr{O}}) = \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \mathfrak{A}|O_{\mathscr{O}} = \mathfrak{N}_{\mathscr{O}} \wedge \mathfrak{A} \vDash \Theta\} \tag{21}$$
$$\mathrm{S}_2(\mathfrak{N}_{\mathscr{O}}) = \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \mathfrak{A}|O_{\mathscr{O}} = \mathfrak{N}_{\mathscr{O}}\} \tag{22}$$

The class $\mathrm{S}(\mathfrak{N}_{\mathscr{O}})$ of *admissible $\mathscr{V}$-structures* for $\Theta$ and $\mathfrak{N}_{\mathscr{O}}$ is defined as

$$\mathrm{S}(\mathfrak{N}_{\mathscr{O}}) := \mathrm{S}_1(\mathfrak{N}_{\mathscr{O}}) \text{ if } \mathrm{S}_1(\mathfrak{N}_{\mathscr{O}}) \neq \varnothing, \quad \mathrm{S}(\mathfrak{N}_{\mathscr{O}}) := \mathrm{S}_2(\mathfrak{N}_{\mathscr{O}}) \text{ if } \mathrm{S}_1(\mathfrak{N}_{\mathscr{O}}) = \varnothing\,. \tag{23}$$

$\mathfrak{A}|O_{\mathscr{O}}$ here stands for the *relativized reduct* of $\mathfrak{A}$, the substructure of $\mathfrak{A}$'s reduct $\mathfrak{A}|_{\mathscr{O}}$ that has domain $O$ (i.e., $\mathfrak{A}|O_{\mathscr{O}} := \mathfrak{A}|_{\mathscr{O}}|O$). It is a standard notion in model theory (cf. Hodges 1993, §5.1).

[26] The quotes from Hempel and Carnap that Suppe adduces to show that they accept direct interpretations of $\mathscr{T}$-terms in fact show the exact opposite (Lutz 2012b, 93–94).

[27] Personal communication from 10 January 2013.

Given this semantics, Andreas defines truth in $\mathfrak{N}_{\mathcal{O}}$ as supertruth in $\mathbf{S}$. Since $\mathbf{S}(\mathfrak{N}_{\mathcal{O}})|O_{\mathcal{O}} := \{\mathfrak{S}|O_{\mathcal{O}} : \mathfrak{S} \in \mathbf{S}(\mathfrak{N}_{\mathcal{O}})\} = \{\mathfrak{N}_{\mathcal{O}}\}$, all $\mathcal{O}$-terms are completely precise over $O$. But as in Przełęcki's semantics (and unlike in Carnap's semantics), most $\mathcal{O}$-terms are completely vague over $U$. This is because for any $\mathfrak{S} \in \mathbf{S}(\mathfrak{N}_{\mathcal{O}})$, any function $f$ such that $f|_O = \mathrm{id}$ and $f|_U = g$ for some permutation $g$ over $U$ leads to another model $\mathfrak{S}'$ of $\Theta$ as described in the proof of claim 3. Since $\mathfrak{S}'|O_{\mathcal{O}} = \mathfrak{N}_{\mathcal{O}}$, $\mathfrak{S}' \in \mathbf{S}(\mathfrak{N}_{\mathcal{O}})$. The only $\mathcal{O}$-terms that are not completely vague over $U$ are those that are invariant under any permutation; such terms may be called purely logical over $U$ (cf. Przełęcki 1969, 30–31). Purely logical terms $R$ occur only if $\mathbf{S}_1 \neq \varnothing$ and $\Theta$ entails sentences like $\forall x\,.\neg Ox \rightarrow Rx$ or $\forall x \forall y\,.Rxy \leftrightarrow x = y$. Note that this exception does not hold for Przełęcki's semantics, since there the extension of the $\mathcal{O}$-terms over $U$ is determined independently of $\Theta$.

Andreas (2010, 538) states that in his account

> only sentences qualifying as postulates are assumed to determine the meaning of theoretical terms. And the distinction between postulates and other theoretical sentences must clearly not be equated with the analytic-synthetic distinction. Analyticity is therefore no requirement for a sentence to determine the meaning of nonlogical symbols.

Although Andreas did not set out developing a semantics that allows a clear analytic-synthetic distinction, I contend that he did. One way to show this is to follow Przełęcki's suggestion of introducing a possibly fictitious new observational vocabulary, and to treat Andreas's $\mathcal{O}$-terms as theoretical terms. Andreas's semantics is then recovered by defining his $\mathcal{O}$-terms conditionally with the help of the new observational terms (Lutz 2012a, §2.10.2). In this way, Andreas's $\mathcal{O}$-terms are completely vague over all unobservable objects as a result of the conditional definitions, as was already suggested by Przełęcki. A more direct way leads via Przełęcki's criteria of analyticity that take previously established analytic sentences into account (definition 11). The discussion in the following will be much simplified by focusing not on the set of sentences $\Theta$ describing a scientific theory, but rather the class $\mathbf{T} := \{\mathfrak{A} : \mathfrak{A} \models \Theta\}$ of its models. Similarly, one can define the analytic component $\mathrm{An}(\mathbf{T}) := \{\mathfrak{A} : \mathfrak{A} \models \mathrm{An}(\Theta)\}$ of $\mathbf{T}$ and the class $\mathbf{A}_{\mathcal{O}} := \{\mathfrak{A}_{\mathcal{O}} : \mathfrak{A}_{\mathcal{O}} \models \Pi_{\mathcal{O}}\}$ of analytic $\mathcal{O}$-structures. As first suggested by Caulton (2012), I will go one step further and treat these these classes as primitive, thus specifically not assuming that they have an axiomatization in first order logic.[28]

Andreas's semantics is intended for *any* $\mathcal{O}$-structure with domain $O$. Thus, while he defines $\mathbf{S}(\mathfrak{N}_{\mathcal{O}})$ as the class of admissible structures for $\Theta$ *and* $\mathfrak{N}_{\mathcal{O}}$, it is of interest to determine which structures are admissible in principle, that is, independently of the empirically determined $\mathfrak{N}_{\mathcal{O}}$.

**Definition 19.** The class of *admissible structures for* $\Theta$ is given by

$$\mathbf{S}^* := \bigcup \{\mathbf{S}(\mathfrak{A}_{\mathcal{O}}) : |\mathfrak{A}_{\mathcal{O}}| = O\} \tag{24}$$

[28] Caulton gives a semantic version of *Carnap's* conditions of adequacy (definition 3).

This can be simplified by using $\mathbf{T}|O_{\mathscr{O}} := \{\mathfrak{T}|O_{\mathscr{O}} : \mathfrak{T} \in \mathbf{T}\}$ and $\exists\mathfrak{A}|O_{\mathscr{O}}$ for the claim that $\mathfrak{A}|O_{\mathscr{O}}$ has a relativized reduct with domain $O$:

**Claim 15.** *The class of admissible structures for* $\Theta$ *is*

$$\mathbf{S}^* = \left\{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \exists\mathfrak{A}|O_{\mathscr{O}} \wedge (\mathfrak{A}|O_{\mathscr{O}} \notin \mathbf{T}|O_{\mathscr{O}} \vee \mathfrak{A} \in \mathbf{T})\right\} \tag{25}$$

*Proof.* First note that every $\mathscr{O}$-structure $\mathfrak{A}_{\mathscr{O}}$ with $|\mathfrak{A}_{\mathscr{O}}| = O$ is in $\mathbf{S}^*|O_{\mathscr{O}}$, either because it can be extended and expanded to a model of $\Theta$ with domain $O \cup U$ (which is then in $\mathbf{S}_1$), or because every such expansion makes $\Theta$ false (and is thus in $\mathbf{S}_2$). Hence the following holds:

$$\begin{aligned} \mathbf{S}^* =& \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \exists\mathfrak{A}|O_{\mathscr{O}} \wedge \mathfrak{A} \in \mathbf{T}\} \\ & \cup \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \exists\mathfrak{A}|O_{\mathscr{O}} \wedge \mathfrak{A}|O_{\mathscr{O}} \notin \mathbf{T}|O_{\mathscr{O}}\} \end{aligned} \tag{26}$$

$$= \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \exists\mathfrak{A}|O_{\mathscr{O}} \wedge (\mathfrak{A}|O_{\mathscr{O}} \notin \mathbf{T}|O_{\mathscr{O}} \vee \mathfrak{A} \in \mathbf{T})\} \tag{27}$$

□

Przełęcki's definition 11 can be rephrased using classes of structures:

**Definition 20.** $\mathrm{An}(\mathbf{T})$ is a *semantically adequate analytic component of* $\mathbf{T}$ *given analytic* $\mathscr{O}$*-structures* $\mathbf{A}_{\mathscr{O}}$ if and only if

$$\forall\mathfrak{A}_{\mathscr{O}} \,.\, \mathfrak{A}_{\mathscr{O}} \in \mathbf{A}_{\mathscr{O}} \rightarrow \exists\mathfrak{B} \,.\, \mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}_{\mathscr{O}} \wedge \mathfrak{B} \in \mathrm{An}(\mathbf{T}) \tag{28}$$

and

$$\forall\mathfrak{A} \,.\, \mathfrak{A}|_{\mathscr{O}} \in \mathbf{A}_{\mathscr{O}} \wedge \exists\mathfrak{B}[\mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}|_{\mathscr{O}} \wedge \mathfrak{B} \in \mathbf{T}] \rightarrow [\mathfrak{A} \in \mathrm{An}(\mathbf{T}) \leftrightarrow \mathfrak{A} \in \mathbf{T}] \,. \tag{29}$$

It is clear that for $\mathbf{T} = \{\mathfrak{A} : \mathfrak{A} \vDash \Theta\}$, $\mathrm{An}(\mathbf{T}) = \{\mathfrak{A} : \mathfrak{A} \vDash \mathrm{An}(\Theta)\}$, and $\mathbf{A}_{\mathscr{O}} = \{\mathfrak{A}_{\mathscr{O}} : \mathfrak{A}_{\mathscr{O}} \vDash \Pi_{\mathscr{O}}\}$, definitions 11 and 20 are equivalent. It is now possible to establish the status of $\mathbf{S}^*$:

**Claim 16.** $\mathbf{S}^*$ *is a semantically adequate analytic component of* $\mathbf{T}$ *given analytic* $\mathscr{O}$*-structures*

$$\mathbf{A}_{\mathscr{O}} := \left\{\mathfrak{A}_{\mathscr{O}} : |\mathfrak{A}_{\mathscr{O}}| = O \cup U \wedge \exists\mathfrak{A}_{\mathscr{O}}|O_{\mathscr{O}} \wedge \left(\mathfrak{A}_{\mathscr{O}}|O \notin \mathbf{T}|O_{\mathscr{O}} \vee \mathfrak{A}_{\mathscr{O}} \in \mathbf{T}|_{\mathscr{O}}\right)\right\} \,. \tag{30}$$

*Proof.* To show that equation (28) holds for $\mathbf{S}^*$, $\mathbf{T}$, and $\mathbf{A}$, assume that $\mathfrak{A}_{\mathscr{O}} \in \mathbf{A}_{\mathscr{O}}$. Then $\mathfrak{A}_{\mathscr{O}}|O \notin \mathbf{T}|O_{\mathscr{O}}$ or $\mathfrak{A}_{\mathscr{O}} \in \mathbf{T}|_{\mathscr{O}}$. In the former case, there is no $\mathfrak{B} \in \mathbf{T}$ with $|\mathfrak{B}| = O \cup U$ such that $\mathfrak{B}|O_{\mathscr{O}} = \mathfrak{A}_{\mathscr{O}}$, and thus any extension and expansion of $\mathfrak{A}_{\mathscr{O}}$ to $O \cup U$ and $\mathscr{T}$ is in $\mathbf{S}^*$. In the latter case, there is such a $\mathfrak{B} \in \mathbf{T}$, and thus, by claim 15, $\mathfrak{B} \in \mathbf{S}^*$.

To show that equation (29) holds, assume that $\mathfrak{A}|_{\mathscr{O}} \in \mathbf{A}_{\mathscr{O}} \wedge \exists\mathfrak{B}[\mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}|_{\mathscr{O}} \wedge \mathfrak{B} \in \mathbf{T}]$. If further $\mathfrak{A} \in \mathbf{T}$, then $\mathfrak{A} \in \mathbf{S}^*$ by claim 15. If on the other hand $\mathfrak{A} \in \mathbf{S}^*$, then by claim 15, $\mathfrak{A}|O_{\mathscr{O}} \notin \mathbf{T}|O_{\mathscr{O}} \vee \mathfrak{A} \in \mathbf{T}$. The first disjunct is false because $\exists\mathfrak{B}[\mathfrak{B}|_{\mathscr{O}} = \mathfrak{A}|_{\mathscr{O}} \wedge \mathfrak{B} \in \mathbf{T}]$, so $\mathfrak{A} \in \mathbf{T}$. □

Claim 16 hinges on the definition of the analytic $\mathcal{O}$-structures $\mathbf{A}_{\mathcal{O}}$, and it is clear that $\mathbf{S}^*$ can always be an adequate analytic component if $\mathbf{A}_{\mathcal{O}}$ is chosen small enough. For instance, if $\mathbf{A}_{\mathcal{O}} = \varnothing$, both conditions in definition 20 are trivially fulfilled. But $\mathbf{A}_{\mathcal{O}}$ is not too strict; more precisely, it does not place any restrictions on the $\mathcal{O}$-structures with domain $O$:

**Claim 17.**

$$\mathbf{A}_{\mathcal{O}}|O = \{\mathfrak{A}_{\mathcal{O}} : |\mathfrak{A}_{\mathcal{O}}| = O\} \tag{31}$$

*Proof.* Let $|\mathfrak{A}_{\mathcal{O}}| = O$. For a proof by cases, assume that for some $\mathfrak{B}$, $\mathfrak{B}|O_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}} \wedge |\mathfrak{B}| = O \cup U \wedge \mathfrak{B} \in \mathbf{T}$. Then $\mathfrak{B}|_{\mathcal{O}}|O = \mathfrak{A}_{\mathcal{O}}$ and $|\mathfrak{B}|_{\mathcal{O}}| = O \cup U \wedge \exists \mathfrak{B}|_{\mathcal{O}}|O_{\mathcal{O}} \wedge \mathfrak{B}|_{\mathcal{O}} \in \mathbf{T}|_{\mathcal{O}}$, and thus $\mathfrak{A}_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}}|O$.

Now assume that there is no $\mathfrak{B}$ with $\mathfrak{B}|O_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}} \wedge |\mathfrak{B}| = O \cup U \wedge \mathfrak{B} \in \mathbf{T}$. Then choose any extension $\mathfrak{B}_{\mathcal{O}}$ of $\mathfrak{A}_{\mathcal{O}}$ to $O \cup U$. By assumption, $\mathfrak{B}_{\mathcal{O}}$ has no expansion that is in $\mathbf{T}$. Thus $|\mathfrak{B}_{\mathcal{O}}| = O \cup U \wedge \exists \mathfrak{B}_{\mathcal{O}}|O_{\mathcal{O}} \wedge \mathfrak{B}_{\mathcal{O}} \notin \mathbf{T}|O_{\mathcal{O}}$, and thus $\mathfrak{A}_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}}|O$.

The proof in the other direction is immediate. □

Since $\mathbf{T}$ has an analytic component, it should also have a synthetic component. The analogue to definition 20 is given by

**Definition 21.** $\mathrm{Syn}(\mathbf{T})$ is a *semantically adequate synthetic component of* $\mathbf{T}$ *given analytic* $\mathcal{O}$*-structures* $\mathbf{A}_{\mathcal{O}}$ if and only if

$$\forall \mathfrak{A}_{\mathcal{O}} \,.\, \mathfrak{A}_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}} \rightarrow \big(\exists \mathfrak{B}[\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}} \wedge \mathfrak{B} \in \mathbf{T}] \leftrightarrow \exists \mathfrak{B}[\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}} \wedge \mathfrak{B} \in \mathrm{Syn}(\mathbf{T})]\big) \tag{32}$$

and

$$\forall \mathfrak{A} \forall \mathfrak{B} \,.\, \mathfrak{A}|_{\mathcal{O}} = \mathfrak{B}|_{\mathcal{O}} \wedge \mathfrak{A}|_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}} \rightarrow [\mathfrak{A} \in \mathrm{Syn}(\mathbf{T}) \leftrightarrow \mathfrak{B} \in \mathrm{Syn}(\mathbf{T})] \,. \tag{33}$$

The first condition generalizes definition 8, and the second condition generalizes definition 9. While $\mathbf{S}$ leads to an analytic component of $\mathbf{T}$, $\mathbf{S}_1$ leads to $\mathbf{T}$'s synthetic component. Define

$$\mathbf{S}_1^* := \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \exists \mathfrak{A}|O_{\mathcal{O}} \wedge \mathbf{S}_1(\mathfrak{A}|O_{\mathcal{O}}) \neq \varnothing\} \tag{34}$$

This can again be simplified:

$$\mathbf{S}_1^* = \{\mathfrak{A} : |\mathfrak{A}| = O \cup U \wedge \exists \mathfrak{A}|O_{\mathcal{O}} \wedge \mathfrak{A}|O_{\mathcal{O}} \in \mathbf{T}|O_{\mathcal{O}}\} \tag{35}$$

The following then holds:

**Claim 18.** $\mathbf{S}_1^*$ *is a semantically adequate synthetic component of* $\mathbf{T}$ *given analytic* $\mathcal{O}$*-structures* $\mathbf{A}_{\mathcal{O}}$.

*Proof.* For a proof of equation (32), assume $\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}}$ and $\mathfrak{B} \in \mathbf{S}_1^*$. Then by equation (35), $\mathfrak{A}_{\mathcal{O}}|O = \mathfrak{B}|O_{\mathcal{O}} \in \mathbf{T}|O_{\mathcal{O}}$. Hence, since $\mathfrak{A}_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}}$, $\mathfrak{A}_{\mathcal{O}} \in \mathbf{T}|_{\mathcal{O}}$, and thus there is some $\mathfrak{B}$ with $\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}_{\mathcal{O}}$ and $\mathfrak{B} \in \mathbf{T}$. The other direction of the equivalence is immediate.

For a proof of equation (33), assume $\mathfrak{B}|_{\mathcal{O}} = \mathfrak{A}|_{\mathcal{O}} \in \mathbf{A}_{\mathcal{O}}$ and $\mathfrak{A} \in \mathbf{S}_1^*$. Then by equation (35), $\mathfrak{B}|O_{\mathcal{O}} = \mathfrak{A}|O_{\mathcal{O}} \in \mathbf{T}|O_{\mathcal{O}}$ and thus $\mathfrak{B} \in \mathbf{S}_1^*$. □

In summary, then, for each theory $\mathbf{T}$ it is possible to find an analytic component and a synthetic component under the assumption that there are already analytic $\mathscr{O}$-structures.

This result might be unsurprising given the way Andreas sets up his semantics. He assumes that any $\mathscr{O}$-structure with domain $O$ can be an intended structure, and then allows any extension to the whole domain $O \cup U$. Thus the empirical information in an $\mathscr{O}$-structure with domain $O \cup U$ is completely contained in its substructure with domain $O$. This means that the extensions of the $\mathscr{O}$-terms over $U$ do not provide empirical, but only analytic information. Thus any restrictions of the extensions over $U$ that follow from $\Theta$ are analytic, and when this is taken into account, Przełęcki's formalism for an analytic-synthetic distinction with previously given analytic structures becomes applicable.

* * *

The overall result so far is this: Przełęcki's semantics anticipates and generalizes concepts and results from the partial structures approach, constructive empiricism and its critics, Suppe's semantics, and Andreas's semantics. Since Suppe's and Andreas's semantics are meant to be elaborations of Carnap's informal remarks, this provides another argument for the adequacy of Przełęcki's formalism as a semantics of the Received View.

## 5 Analyticity in vague languages

There is no question that most if not all terms are vague. This holds on empirical grounds for $\mathscr{O}$-terms and, due to Winnie's argument, especially for $\mathscr{T}$-terms over $U$. Since Przełęcki assumes that the intended $\mathscr{O}$-structures $\mathbf{N}_{\mathscr{O}}$ are determined solely by ostensive interpretation and that the negative extension of a term is determined analogously to the positive extension of a term (by comparison with the negative standards), he can conclude that $\mathscr{O}$-terms are completely vague over the unobservable objects. This is in contrast to Carnap's assumption that the $\mathscr{O}$-terms are precise over $U$, simply because their extension is disjoint from $U$ (and hence the permutation argument given in connection with Andreas's semantics does not apply). Whether Carnap's (and Suppe's) or Przełęcki's (and Andreas's) view on the interpretation of $\mathscr{O}$-terms is more apt depends on whether the negative extension of a term can *only* be determined by comparison with the negative standards. I at least doubt that this is in general the case, since, for example, I expect that the positive standards of 'blue' are sufficient do definitely exclude a kiss on the neck from its extension.[29] In any case, nothing very important hinges on the decision: To move from Przełęcki's view to Carnap's, every $\mathscr{O}$-term may

[29]While this is an empirical question in the case of observable objects, things are somewhat different for unobservable objects since these cannot even be referred to by ostension. Hence one cannot ask whether 'this' is red by pointing to it or, in the case of the kiss, performing it. I will have to leave this line of thought for another time.

be substituted by a new $\mathcal{O}$-term that is co-extensional except in that its negative extension includes $U$. To move from Carnap's view to Przełęcki's, every $\mathcal{O}$-term may substituted by a new $\mathcal{O}$-term that is co-extensional except that its neutral extension includes $U$.

The move from Przełęcki's view to Carnap's introduces analytic $\mathcal{O}$-sentences, which state for every $\mathcal{O}$-term that it does not apply to unobservable objects. The move from Carnap's view to Przełęcki's does not seem to introduce any such statements, since it only restricts the positive and negative extensions. Therefore the new set of intended interpretations $\mathbf{N}_{\mathcal{O}}$ is a superset of the old set, and thus renders at most as many sentences true and at most as many sentences false as the old. In that sense, Przełęcki's view may seem preferable for allowing the assumption of $\mathcal{O}$-terms without analytic sentences. But it does not allow this; specifically, languages with vague $\mathcal{O}$-terms sometimes entail the existence of analytic $\mathcal{O}$-sentences.

Andreas's semantics provides one instance of this phenomenon: Because the intended structures do not fix the extensions of the intended $\mathcal{O}$-structures to $U$, any such restriction by $\Theta$ becomes non-empirical and thus analytic. In the discussion above, this is expressed by the claim that $\mathbf{A}_{\mathcal{O}}$ is not the set of all $\mathcal{O}$-structures over $O \cup U$, so typically not all analytic sentences are tautological. More generally, whenever $\Omega := \{\omega : \mathbf{N}_{\mathcal{O}} \vDash \omega\}$ is not maximally consistent, there will be an $\mathcal{O}$-sentence that is analytic, since its truth value is not determined by $\mathbf{N}_{\mathcal{O}}$, which, by assumption, contains all empirical information there is. Only if a language is too weak to distinguish between at least some of the elements of $\mathbf{N}_{\mathcal{O}}$ do the vague terms fail to lead to analytic sentences. This result has the interesting corollary that Carnap's use of the Ramsey sentence as the synthetic component of a theory is generally justified only if all $\mathcal{O}$-terms are completely precise, or at least so precise that even a higher order sentence cannot distinguish between their different extensions. Otherwise, some theories' Ramsey sentences would have analytic components, which is impossible by assumption. Thus Carnap's use of the Ramsey sentence is incompatible with Andreas's and Przełęcki's semantics, which suggests that Carnap indeed assumed that unobservable objects are in the negative extension of all $\mathcal{O}$-terms.

While the analytic sentences in Andreas's semantic are easily determined, the general case suggests an interesting and possibly deep puzzle. $\mathbf{N}_{\mathcal{O}}$ is assumed to be determined by language and world together, and the argument just given shows that most $\mathbf{N}_{\mathcal{O}}$ of vague languages lead to analytic $\mathcal{O}$-sentences. But since $\mathbf{N}_{\mathcal{O}}$ is in part determined by the world, there is no guarantee that the analytic $\mathcal{O}$-sentences will be the same for all worlds. Thus an $\mathcal{O}$-sentence may be clearly analytic, but that it is clearly analytic may be an empirical fact.

## 6 The semantics of scientific theories

Przełęcki's formalism fulfills the central desiderata for a successful explication of the semantics of the Received View. For one, it is broadly compatible with Hempel's and especially Carnap's elaborations of the Received View. This is shown by its equivalence or near-equivalence with Suppe's and Andreas's semantics, which are meant to explicate the Received View as well. More importantly, however, Przełęcki's semantics provides a rigorous framework for many discussions among philosophers of science, including Putnam, French and DaCosta, Demopoulos, Ketland, and, in part, van Fraassen. Finally, and most significantly, Przełęcki's semantics suggests the further investigation of vagueness, approximation, Ramsey and Carnap sentences, analyticity, and much more that I have not touched on in this discussion. Hence there are good reasons to consider Przełęcki's semantics the best we have for scientific theories, and, I would argue, for large parts of philosophy in general.

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